Demystifying Alpha Go

by

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B.Sc., BRAC University, 2015

AN ESSAY SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Science

in

THE FACULTY OF GRADUATE AND POSTDOCTORAL

STUDIES

(Computer Science)

The University of British Columbia

(Vancouver)

April 2018

Abstract

While Go programs have been around since the 1970s, their performance has not come close to achieving professional level playing let alone defeating Go champions. The team of researchers at DeepMind were able to tackle this daunting challenge with their Go program: AlphaGo. AlphaGo's journey towards achieving superhuman-level performance in Go came into the spotlight in 2016 when it defeated Go world champion Lee Sedol. While it is easy to be carried away by the enormous hype surrounding AlphaGo's achievement, it is a worthwhile exercise to delve deeper into how AlphaGo works and in the process demystify some of the hype surrounding this truly extraordinary achievement. This is the primary motivation of this essay. By looking deeper into AlphaGo's architecture, we can see that it is indeed a combination of techniques, some of which have been developed by researchers in the early days of computer Go, aided with state of the art hardware.

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Acknowledgements

I would like to thank my academic advisor Prof. David Poole for helping me find an interesting and worthwhile topic for my essay. The primary objective of this essay is to demystify the underlying mechanisms behind AlphaGo, a worthy endeavor given the enormous hype surrounding it ever since AlphaGo defeated Lee Sedol in 2016. I'm also grateful for having the opportunity to take multiple courses in artificial intelligence and machine learning at UBC which has helped me attain the necessary background to appreciate this topic.

Chapter 1

Introduction

Since the inception of AI, classical games have been studied as application fields [1]. By the early 2000s, researchers had already created computer programs that achieved superhuman performance for a variety of games including chess and checkers. While games such as chess, checkers and Go are two-player games of perfect information, a Go program that can surpass the best human Go players was still a feat beyond the capabilities of AI researchers till 2016. What makes Go particularly challenging is the intractable search space of 10^{170} possible board positions which meant that it was infeasible to solve using approaches used in other games [1]. While computer Go programs have been around since the 1970s, their performance had not come close to achieving professional level playing let alone defeating Go champions [1]. The team of researchers at DeepMind were able to tackle this daunting challenge with their Go program: AlphaGo [2]. AlphaGo's journey towards achieving superhuman-level performance in Go came into the spotlight in April 2016 when it defeated Go world champion Lee Sedol in a five-game match. In May 2017, AlphaGo's reputation as the first Go program to truly reach superhuman-level performance was bolstered even more when it defeated Ke Jie, the top ranked professional Go player at the time [3].

While it is easy to be carried away by the hype surrounding AlphaGo's achievement, it is a worthwhile exercise to delve deeper into how AlphaGo works and demystify some of the hype surrounding this truly extraordinary achievement. This is the primary motivation of this essay. By looking deeper into AlphaGo's architecture, we can see that it is indeed a combination of techniques, some of which have been developed by researchers in the early days of computer Go, aided with state of the art hardware.

The remainder of the essay is organized as follows: Chapter 2 starts out by providing the necessary background to understand AlphaGo and Go programs in general: it gives a brief overview of the game's rules, why it is challenging, how computer Go has evolved since its inception, and ends with a discussion on why AlphaGo is revolutionary. Chapter 3 delves into the underlying techniques used by AlphaGo and a discussion on how AlphaGo successfully combines previous research into a revolutionary Go program. Chapter 4 ends the essay by providing concluding thoughts and a look at AlphaGo’s potential impact on computer Go research and beyond.

Chapter 2

Background

2.1 The Game of Go

Go is played on a $19x19$ board with two players. One player plays using white stones (let's call him *player 1*) while the other plays black stones (let's call her *player 2*). The board is initially empty; player 1 starts the game by placing one of his white stones in one of the grid intersections. This is followed by player 2 placing one of her black stones in an unoccupied grid point. Players are not allowed to move the stones once they are placed on the board. The game continues by players taking turns to put their respective stones in previously unoccupied positions on the board. A player can opt to skip a move, allowing the other player to effectively play two moves at once.

The goal of each player is to maximize their territory i.e. place as many stones on the board as possible. Each stone has *liberties* i.e. free adjacent cells. The adjacent stones of the same color form a *group*. A player can capture a group of the opponent's stones by surrounding it with his or her own stones. This results in the removal of the whole group from the game board.

The game concludes when both players have passed i.e. skipped their turns consecutively. A player withdrawing will also result in ending the game. The scores are then counted. While the system of counting points differs, the most common system involves awarding points to $player\ i$ proportionally to the number of grid points occupied by i. Martin Müller provides a detailed look at the rules [4].

2.2 Why is Go challenging?

Before we discuss why Go is challenging to solve computationally, let us start by talking about why classical games like chess and Go are well suited to computational solutions. Go is a sequential game of *perfect information*. This means that before making a move, each player is perfectly informed of all events that have previously occurred [5]. More precisely, each player can see all the stones on the board throughout the duration of the game. This means that the outcome of the game is solely determined by the strategy of the two players [6]. This fact makes Go an attractive problem to solve computationally - we simply need to design a program that solves for the optimal sequence of moves.

One way to understand why Go is challenging to solve computationally, is to start by looking at two other classical games that are also of the sequential/perfect information variety. Chess and Checkers are two popular sequential games of perfect information which have achieved superhuman performance using computational solutions. Checkers and Chess have search spaces of 10^{20} and 10^{43} possible positions respectively [5, 7]. In comparison, Go has a search space of 10^{170} possible positions [8, 9], which renders exhaustive search infeasible. In other words, since Go has a large number of potential moves, it yields exponentially more ways for the game to unfold relative to chess and checkers [5]. Table 1 illustrates how Checkers and Chess have much smaller branching factors (number of legal moves per position) and average game depths (length of the game) relative to Go.

Game	**Branching Factor (b)**	**Game Depth (d)**
Chess [7]	35	80
Checkers [8]	2.8	70
Go [7]	250	150

Table 1: Complexity of different sequential games of perfect information

With a *branching factor* of 250 and an average *game depth* of 150, exhaustive search is computationally infeasible for Go. This daunting search space had meant that achieving superhuman performance in Go was regarded unsolvable in the near future by most researchers [2, 5]. Another reason why Go is challenging to solve computationally is because it is difficult to accurately predict which player is more likely to win the game from any given board position. This limits the efficacy of simply applying machine learning approaches [5].

2.3 History and Evolution of Computer Go

The first scientific paper about Computer Go was published in 1963, where it considered the possibility of applying machine learning in Go [1, 10]. The first recorded instance of a Go program to defeat a human player, albeit, an absolute beginner was developed by Zobrist in 1970 [11, 12]. It was primarily based on the computation of a potential function that approximated the influence of stones [1]. Since the beginning of this field of study, researchers have been working on sub-problems of the game, namely working with smaller boards or localized problems like life and death of groups [1]. Computer Go became an active research field in the 1980s, when the first journal devoted to Computer Go was issued, in addition to early releases of several commercial programs [1].

In 1979, the first Go program to play at a level above a human beginner was designed by Reitman and Wilcox [13]. Wilcox's approach used abstract representations of the game board and reasoned about groups [1, 13]. The next notable breakthrough was by Boon in 1990 to use patterns to recognize common situations and to suggest moves [14].

The first instance of a Go program leveraging the Monte Carlo method was Gobble in 1993 [15]. By using the average of numerous simulations, Gobble assigned approximate values to possible moves; then it considered only the moves with the highest values. However, Gobble fared quite poorly relative to human Go players – it achieved a rank of only 25 kyu which translates to a very weak beginner [2, 15].

In 2006, Crazy Stone [16] became one of the first Go programs to incorporate Monte Carlo Tree Search (MCTS). It managed to outperform most of its contemporary competition which made use of conventional techniques. Since then, MCTS has been a staple in the computer Go community, eventually forming a critical part of AlphaGo's architecture [2].

Convolutional Neural Networks (CNNs) have been successfully used in Computer Go several times before AlphaGo [8, 17, 18]. DeepMind's AlphaGo successfully combined MTCS with CNNs to develop the first program to achieve superhuman performance [2].

2.4 The Revolution: AlphaGo

The team of researchers at DeepMind were able to overcome this daunting challenge with AlphaGo. AlphaGo's journey towards achieving superhuman-level performance in Go came into the spotlight in April 2016 when it defeated Go world champion Lee Sedol in a five-game match.

So, what makes AlphaGo so special relative to other Go programs? While the hype surrounding AlphaGo's achievement made it appear like sorcery, looking deeper into DeepMind's work [2], we can see that it is indeed to a large extent, a blend of tried and tested techniques, some of which have been developed by researchers in the early days of computer Go, aided with state of the art hardware. The first instance of a Go program leveraging the Monte Carlo method was Gobble in 1993 [15]. This was followed by Crazy Stone which was the first Go program to incorporate MCTS which resulted in it outperforming most of its contemporary competition [16]. Several other groups [8, 17, 18] leveraged Go's translational invariance to successfully apply CNNs in Go. These important research contributions preceding AlphaGo laid the groundwork for the creation of AlphaGo. This is by no means a criticism of DeepMind's work, but a mere acknowledgement of the fact that AlphaGo's success is built on important contributions by the computer Go research community. While many of these techniques may not be novel in Computer Go, what makes AlphaGo truly revolutionary is the combination of these techniques i.e. their ensemble. We explore these techniques in Section 3.

Chapter 3

A deeper look at AlphaGo

3.1 Monte Carlo Tree Search (MCTS)

Monte Carlo methods are a class of computational algorithms that rely on repeated random sampling to obtain numerical results [19]. The main idea is to use randomness to solve problems that are deterministic in principle, but too complex to solve using other approaches e.g. when a problem is too complex to be solved analytically [5].

An efficient way to solve the game search tree is to apply the Monte Carlo Method. This approach approximates a function using random sampling, where the mean of the samples converges to the real value of a function [5]. It is a selective search technique that leverages prior estimations to select the most promising move to explore [18]. By combining game domain knowledge and pattern matching, it tries to model the way humans make moves i.e. by only considering the most profitable moves [18].

The idea to integrate Monte Carlo simulations into the tree growing process, also known as Monte Carlo Tree Search (MCTS) was first adopted by the Go program Crazy Stone [16]. Ever since, this technique has become a staple in the computer Go community.

In MCTS, each move is selected according to its value. This value is accumulated by making random simulations, where each simulation represents a complete game. This approximation is justified by the central limit theorem, which states that the Monte Carlo values (mean of all possible outcomes) converge to the normal distribution [19]. If the tree is explored to a fair extent, the strategy using MCTS converges to the optimal strategy [19]. MCTS comprises of four main stages [20] that are applied sequentially. This is illustrated in Figure 1. The values inside each node are in the form: $\#wins/\ \#visits$.

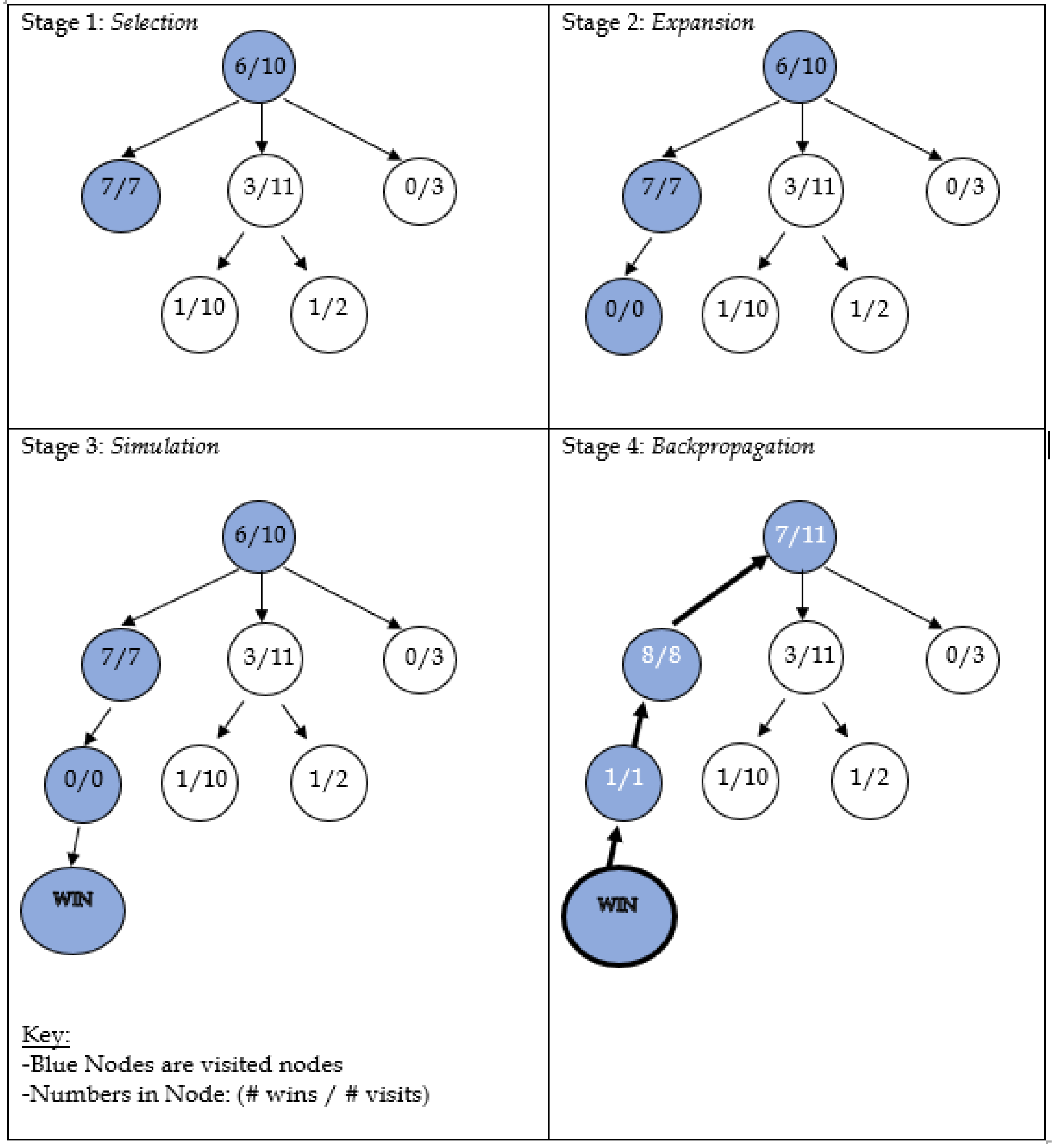


Figure 1: Stages of Monte Carlo Tree Search (MCTS).

3.2 Advantages and disadvantages of using MCTS

Let's start by discussing the benefits of using MCTS:

1. Using MCTS significantly reduces the number of explored nodes in the game tree. By only allowing promising branches, the tree grows asymmetrically with a low branching factor (relative to classical methods like alpha-beta pruning and iterative deepening search) [2, 5].
2. MCTS is well suited to parallelization since the simulations are independent. This property was leveraged by AlphaGo.

However, there are a few disadvantages:

1. MCTS is still not strong enough on its own to overcome the computational challenge of Go. Since the thinking time for the program is limited, using MCTS alone would not allow the program to gather enough statistics to precisely approximate the optimal values for the nodes [5].
2. Random rollouts used in the Simulation phase of MCTS may be unrealistic and hence provide a poor approximation of the target function [5].
3. While the Monte Carlo method has the elegant property that the mean value of all simulations converges to the true value, the only way to guarantee this property is to have an infinite number of simulations [19].

3.3 Augmenting MCTS with Convolutional Neural Nets

Since MCTS on its own is not powerful enough to achieve the performance required, DeepMind augmented it with convolutional neural networks. Let's step back for a moment and try to understand the motivation behind this approach.

As we saw in previous section, the mean value of simulations in MCTS only converges to the true value if it is provided an infinite number of simulations. This is certainly not possible in any domain, especially not in board games like Go where thinking time is limited [5, 19]. Thus, with a limited number of simulations, the quality of each simulation becomes much more important. The higher the quality of each simulation, the faster the convergence towards the optimal sequence of moves [5]. Unlike MCTS, the best human players select a move and try to predict the outcome of the game after playing that move. There is a degree of intuition that the best human players make use of to guide their decision making [18].

To understand the role of CNNs in AlphaGo, we need to first discuss two concepts: *local receptive fields* and *weight sharing*. The concept of *local receptive field* means that each neuron is connected only to a small region of a fixed size in the previous layer [5]. The *weight sharing* assumption is: while every neuron is responsible to its own subarea in the previous layer, the set of weights of connections (i.e. *filter*) is the same for all neurons. We can think of each filter as a feature extractor and the more filters we apply, the more information we can obtain from the input. In AlphaGo, 256 filters were used [2]. The learning task for the CNN was to obtain the weights within each filter [5].

CNNs have been successfully used in several computer vision applications, notably in character recognition and image classification [5]. They operate by extracting local features from the input. While previous work [8, 17, 18] has shown that CNNs are particularly useful for Go due to the translational invariance (evaluation of a position is not affected by shifting the board), the AlphaGo team argued that it hurt performance in large networks as it prevents the intermediate filters from identifying specific patterns. Instead, they exploited symmetries at run-time by dynamically transforming the group of 8 reflections and rotations [2]. For the value network (See *section 3.5,* 2nd paragraph), the output values are simply averaged. For the policy networks, the planes of output probabilities are rotated/reflected back to their respective original positions and then averaged together to provide an ensemble prediction [2].

We can think of the role played by CNNs in AlphaGo as somewhat like the evaluation function used by IBM's Deep Blue [26], one notable distinction being the evaluation function for AlphaGo is learned and not designed or hand crafted [21]. The use of convolutional neural networks in AlphaGo provides a degree of intuition to the game-play.

3.4 Supervised Learning Policy Networks

AlphaGo uses CNNs to predict human moves. Therefore, it makes sense that the input to AlphaGo's CNN is the current board setting [2] and the output is the prediction – the move a human would make. The team at DeepMind used 160,000 games of Go professionals that were recorded on the KGS Go server [2, 23]. Random board positions in conjunction with associated player moves were selected from each game. The network's objective was to predict these actions.

The input to the CNN i.e. the current board setting was translated into 48 features. These features included important information like the color of the stone at each intersection and number of free adjacent cells (liberty) [2]. The selection of these features was influenced by prior work done in 2015 by Clark and Storkey [18]. To summarize the input layer of the CNN was a $19x19x48$ vector, which stored the value of every feature for each intersection of the game board.

The output layer was $19x19$, where each cell indicated the probability that a human player will put a stone in the corresponding intersection of the game board. This is the supervised learning portion of AlphaGo's training. The supervised learning network is used at the selection stage of MCTS to encourage exploitation [2, 5]. A good selection rule tries to find a happy medium between selecting known optimal moves and investigating unexplored moves. We can think of the CNN influencing MCTS to try out moves which have been scarcely explored but which seem good to the CNN [5].

3.5 Reinforcement Learning Policy and Value Networks

Predicting human moves was never the end goal. It was to optimize the networks for maximizing the likelihood of winning and eventually reach superhuman level playing ability. This is where Deep Reinforcement Learning comes into play. The term deep here indicates that the networks being trained have multiple layers. The system plays against itself, without training sets of games played by humans. This approach of self-play is not novel; it was popularized in the reinforcement learning community by Gerald Tesauro in the early nineties when he used it in TD-Gammon [22]. TD-Gammon reached the performance of the top human backgammon players by using this approach of self-play. Reinforcement learning [24] is a method of training an agent where it is not explicitly given the correct answer; instead the agent's object is to maximize its reward. In the context of AlphaGo, the reward used was ± 1 for a win/loss upon termination of the game, and 0 for all other intermediate steps [2]. The idea was that by letting AlphaGo play against other versions of itself, it can learn and improve on its previous performance. Specifically, at a given iteration step, the current version of AlphaGo plays against a random instance of the supervised learning network from the previous iterations. The match ends and a reward is given, which tells the supervised learning network if its actions were correct and how the parameters of the network should be altered [2]. Upon completion of 500 iterations, the version of AlphaGo with the current parameters is added to the opponent pool (this is the collection of previous versions of AlphaGo). In comparison to the supervised learning phase where 160,000 games were used to train the networks, the reinforcement learning network was trained for 10,000 batches of 128 games each, using 50 GPUs for one day [2]. AlphaGo's supervised learning network was greatly strengthened using reinforcement learning here. While the reinforcement learning networks were stronger compared to the supervised learning networks, the best performance was achieved combining the two [2].

The value network has the same architecture as the supervised learning network described in Section 3.4, with the distinction that its job is: given a position in the game to output a single value indicating a win or loss. Additionally, the value network is not trained on human expert games like the supervised learning network; it instead uses the reinforcement learning network's games. This idea of approximating the value function by using reinforcement learning was used earlier by Michael Littman [25].

3.6 Guiding the Simulation Stage

As mentioned in Section 3.1, random simulations from the new node are run in stage 3 of MCTS. Each simulation here represents a complete game. The quality of rollouts or simulations can significantly bolster MCTS [2]. Using a linear classifier that is trained on KGS matches, AlphaGo guides the simulations as follows:

1. A small area surrounding each legal move is selected and compared to precomputed patterns.
2. The input to the linear classifier is the array of indicators that indicate whether the selected area matches any precomputed patters.
3. The output is the probability of that move being selected.

3.7 How AlphaGo performed without search

AlphaGo's performance was tested based purely on policy networks against Pachi, the strongest open-source Go program. It is important to note that Pachi relies extensively on search Specifically, Pachi is a Go program which relies on 100,000 simulations of MCTS at each turn.

At each move, the AlphaGo team selected the actions predicted by the policy networks that gave the maximum likelihood of a win. AlphaGo (based on only its policy networks) won 85% of the games it played against Pachi [2, 21]. This was a truly remarkable result since Pachi relies extensively on search and it got outperformed by AlphaGo's policy networks which relied on Convolutional Neural Networks. This highlights an important property of game play in Go: intuition is very important relative to long reflections [21].

3.8 Summary of AlphaGo's architecture

To summarize, AlphaGo relies on a combination of MCTS and CNNs to guide the tree search procedure.

We can broadly classify AlphaGo's architecture into two distinct phases: (1) Learning Offline and (2) Playing Online.

Phase 1 – Learning Offline

The first phase is an offline phase where it essentially learns to play the game. This is done by training the neural networks. The output of this phase is the supervised learning, reinforcement learning and value networks. Figure 2 illustrates a flowchart for Phase 1 of AlphaGo's architecture.

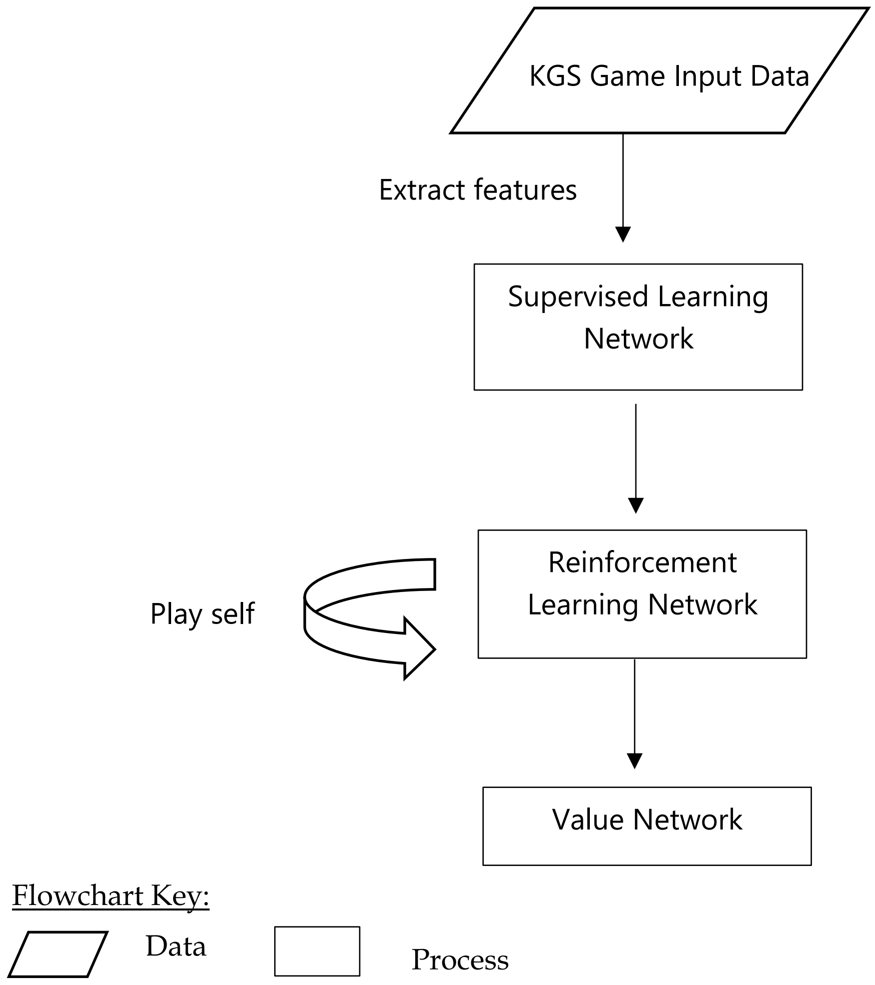


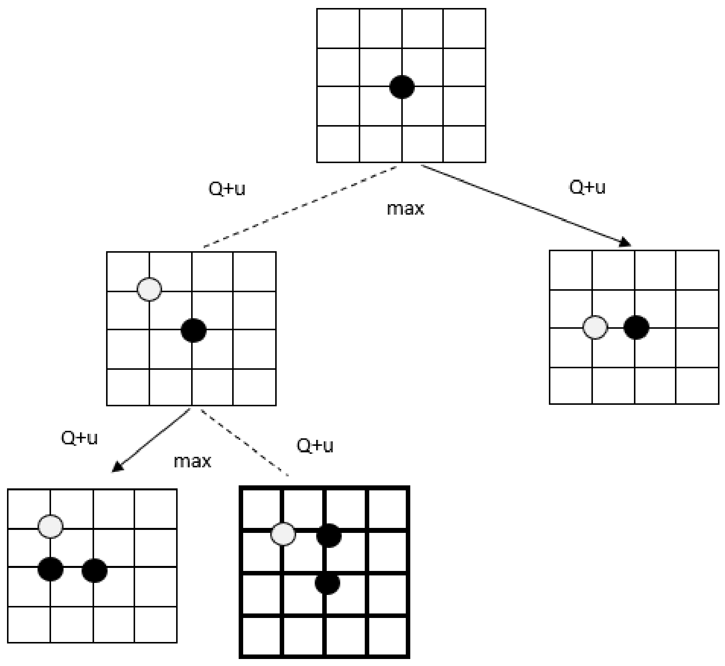
Figure 2: AlphaGo architecture Phase 1 – Offline Learning Flowchart

Phase 2 – Playing Online

The second phase is an online phase where it plays the game by traversing the game tree using MCTS. Figure 3 illustrates Phase 2 of AlphaGo's architecture.

Edges are selected, which contain information regarding the associated nodes. Specifically, each edge stores (1) an action value, $Q\ (node, action)$, (2) a visit count $N(node, action)$ and (3) prior probability $P(node, action)$. The game tree is traversed by descending the tree in complete games (simulations), starting from the root node.

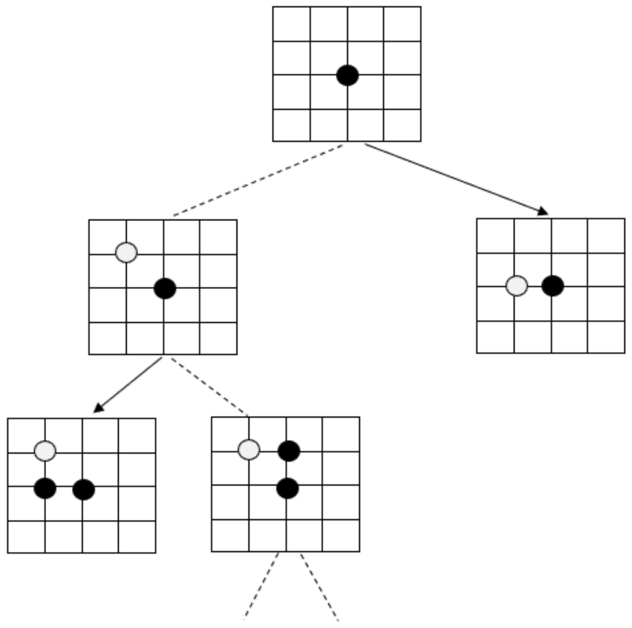
In stage 1 (**selection)** of MCTS, the edge with the maximum action value Q, plus a bonus u is selected. The bonus u depends on the stored prior probability from the supervised learning network. Each edge of the search tree stores an action value Q and visit count N and prior probability P. The dotted lines indicate the path taken.



The dashed lines indicate the path or edges connecting the source node to the selected node.

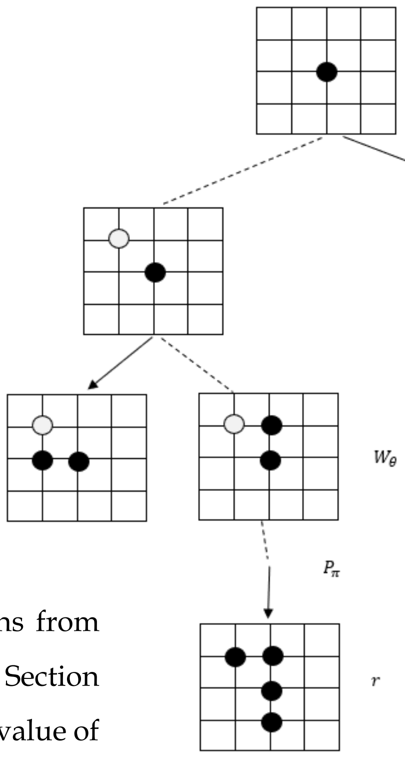
The **bolded node** at the bottom of the game tree indicates the selected node.

In stage 2 (**expansion**), the selected node is added to the game tree and the output probabilities are stored as prior probabilities for each action.



How are the nodes expanded?

Each simulation traverses the game tree by selecting the edge with maximum action value Q, plus a bonus u that depends on a stored prior probability P for that edge. The new node is processed once by the policy network and the output probabilities are stored as prior probabilities P for each action.



In stage 3 (**simulation**), the simulations from the newly selected node are guided (See Section 3.6) by the policy rollout classifier. The value of Q is computed by both the rollout classifier and the value network W_θ.

In other words, once stage 3 is complete (simulation is run), the action value Q and visit counts N are updated for all traversed edges. Once the search is complete, the most visited move from the root position is chosen.

In stage 4 (**backpropagation**), the result of the simulation is then backpropagated i.e. returned to all the nodes in the path. This is denoted by the blue reverse arrows.

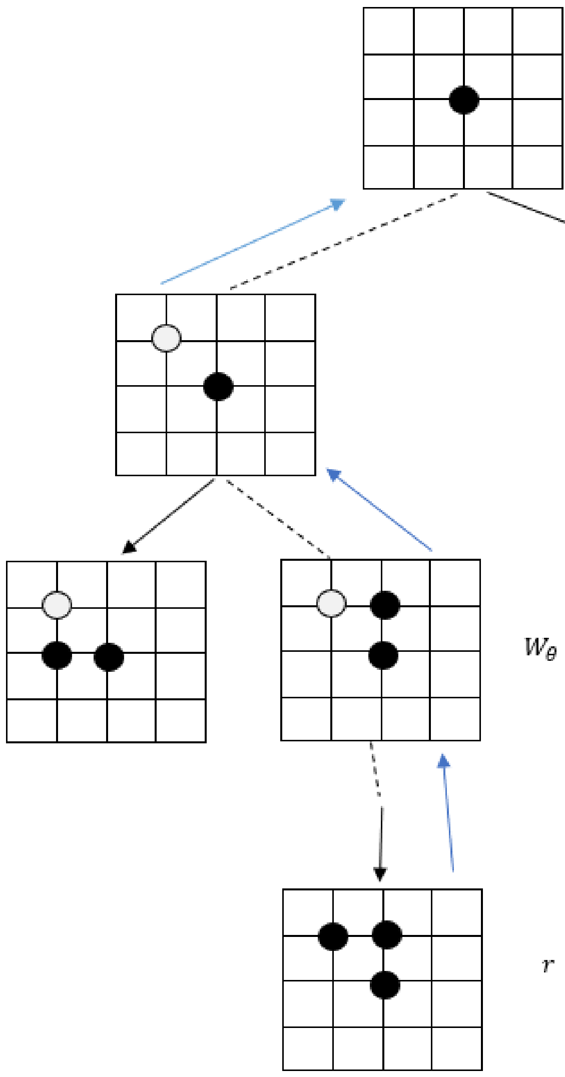


Figure 3: AlphaGo architecture Phase 2 – Online Playing

Chapter 4

Concluding thoughts and future directions

Go was considered the holy-grail of game-playing problems that we did not expect to be solved so quickly. The fact that AlphaGo defeated Lee Sedol is a truly remarkable achievement. If we step back and try to look at the bigger picture, the win in and of itself is not the truly remarkable part. The approach used to win by the AlphaGo team is in my opinion much more important. By making extensive use of both supervised and reinforcement learning instead of using hand-crafted heuristics bodes well for the future. The hope is that this approach can be generalized to solve other problems in AI.

However, a common criticism was that the supervised learning of AlphaGo's training involved training on a large dataset of games played by human players. One could argue that this had made AlphaGo biased to a certain degree towards imitating human play. Maybe if AlphaGo relied completely on self-play instead of having a supervised learning component, new strategies not popular among human Go players may have surfaced. In late 2017, DeepMind tackled this criticism by introducing AlphaGo Zero [27], the next iteration of AlphaGo that relied solely on reinforcement learning. Rather than training on thousands of human games to learn to play Go, AlphaGo Zero skips this step and learns only through self-play. It rapidly surpassed human level of play and defeated the previous champion Go program AlphaGo 100 games to 0 [27]. Interestingly, the new version is simpler: it combines the originally separate policy and value networks used in AlphaGo into one network allowing for more efficient training and evaluation. It also no longer uses rollouts (random simulations previously used to predict which player will win from the current board position). These key differences not only make AlphaGo Zero the new best computer Go program, it also makes it more general. However, there are still a few games that AI hasn't mastered: for example complex strategy games like StarCraft, where humans still reign superior [28].

AlphaGo's success has certainly paved the way for the AI community to tackle interesting new problems. The fact that most experts did not expect to see Go solvers reach this level of performance so quickly due to its intractable search space means that problems that we did not think of being solvable by AI may no longer be beyond us. While AlphaGo Zero is still in its infancy, researchers are optimistic that similar techniques may be applied to other structured problems like protein folding and have a positive impact on society [29].

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